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ON THE RELATION OF OPERATIONS RESEARCH TO MATHEMATICS

George B. Dantzig

Stanford University

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BY

GEORGE B. DANTZIG

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Operations Research should be viewed by mathematicians as a revitalizing force. A fertile ground for new and unsolved problems and for developing new mathematical theories. Universities should be encouraged to develop a Division of Mathematical Sciences encompassing mathematics, operations research, statistics, computer science, and classified applied mathematics. Curricula should be revised so that all mathematics students get a good exposure to the mathematical sciences. The practice of some mathematicians to downgrade the mathematical sciences should be discouraged because: it represents, on their part, ignorance, and lack of responsibility. It deprives the students of an exciting and relevant outlet for their talents.

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This paper was presented as part of a panel talk before The American Mathematics Society, in Atlantic City, January 23, 1971.

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II

PANEL TALK BEFORE

THE AMERICAN MATHEMATICS SOCIETY

Atlantic City, January 23, 1971

by

George B. Dantzig

ON THE RELATION OF OPERATIONS RESEARCH TO MATHEMATICS*

Operations Research should, and I emphasize should, be viewed by Mathematicians as a revitalizing force. A fertile ground for new, unsolved problems, for classes of problems upon which to build new theories. Undergraduate texts should be riddled with an exciting new class of problems which could serve to stimulate and to motivate the student. I would like to build my comments around a combinatorial analysis course I teach at Stanford. This course carries formally a Computer Science label and is cross listed in the Mathematics Department Catalogue. It is an undergraduate course which graduate students can take as an elective. There are a number of good texts that can be used. For example the book by Marshall Hall, or by Herbert Ryser, or by Riordan, or for certain topics the one by Saaty-Busacker is appropriate.

Broadly speaking combinatorial analysis is now taught in two parts which I will label: The first classical, the second important. Classical combinatorics discusses such interesting problems as: In how many ways can I change a five dollar bill? It used to be a one dollar bill but now with inflation--. Classical combinatorics is concerned

*Other panel discussants: Julia Keilson, University of Rochester
Thomas Saaty, University of Pennsylvania
Philip Wolfe, IBM Research
Donald Iglehart, Stanford University, Moderator

with counting problems. Another example: how many ways can a party of 5 men and 5 women sit at a table so that no two men or women are adjacent to each other and no man sits next to his wife? As a mathematician, I like classical combinatorics. It is full of interesting devices: permutations, combinations, generating functions, amusing identities, etc. Relevant, it is not, except as a possible supplement to a basic course in probability. Good courses in probability theory and stochastic processes are, as you know, prerequisites for queueing theory, reliability theory, dynamic programming stochastic control theory, inventory theory. Classical combinatorics is sometimes useful in preventing people from using an exhaustive procedure on the computer such as listing all combinations or examining all the cases. For example, one would probably decide not to list out all the ways to change a 5 dollar bill if he knew in advance there were over 10^9 ways.

The part of combinatorial analysis which I have labeled "important" is concerned with selecting the best combination out of all the combinations.

This is what linear programming is all about. An economy has many alternative technologies it can draw upon. Some use more labor than others, others make more intense use of scarce resources, or more intensive use of limited capacity. The problem becomes one of how to select, how much, and when.

Given a road map, find the shortest path from L.A. to Atlantic City. From an abstract mathematical view one abstracts the cities as a finite set S of points; one abstracts the road connecting two cities as a binary relation between certain pairs of elements of S. These binary relations define a finite graph. A distance function is next assigned to arcs; concepts such as connectedness, paths, trees are then introduced.

Since the number of paths are obviously finite, the problem is obviously uninteresting from a mathematical point of view. Just pick the shortest path among a finite set of paths and you are done. Many mathematicians would label the problem uninteresting because the question of the existence of solution and the method of solution are both obvious. They would also label those who work on such problems as not real mathematicians and the mathematics involved as trivial. Fortunately, not all mathematicians agree with this point of view. For example you the audience don't agree or you would not be here.

A recent AMS panel, similar to this one examined the relation of Computer Science and Mathematics. We are all aware of the computer revolution. Essentially the programs and data we feed into the computer models or mirrors or simulates the real world. We are all aware, barring some utter stupidity in Washington, that research in miniaturization will continue until the day that memory units reach the molecular level or information is stored on light beams in space. Operations Research is concerned, in general, with studying the mathematical properties of the various models that have been proposed for solution on the computer with the view of evolving effecient algorithms. For the most part OR source problems have restricted themselves to decision problems. For this reason OR is sometimes referred to as the "Mathematics of the Decision Sciences."

Let me touch on a semantic difficulty. Properly labeled, the OR field should be called "Applied Mathematics." Unfortunately traditional applied mathematics has as its source, mathematical problems based on the physical sciences. But, the computer revolution has so broadened the base of mathematical modelling that whatever subject you name, is

being described mathematically and put into the computer. Now many mathematicians, as you know, hog the name "Applied Mathematics" and use it to mean only the mathematics of physical science models. In my opinion they are doing the entire field of applicational mathematics a great disservice. The field has long outgrown its tie to physical science. I dare say that there is more progress in mathematical programming alone in one month than progress in traditional applied math. in years. By hogging the term, it has left the choice of an alternative semantic term awkward. As a result Operations Research, Management Science, Control Theory, Numerical Analysis aspects of Computer Science, Statistics don't have a good general descriptive term and neither does traditional applied mathematics. What is emerging, however, is an ever broader descriptive term namely the term "Mathematical Sciences" which, of course, includes mathematics itself.

I recently surveyed my combinatorial analysis course to find out how many had ever heard of operations research, or linear programming, or mathematical programming network theory, integer programming. Only about 1/2 the class did. The rest were mathematics majors. They had been thoroughly indoctrinated by the Hardy syndrome that applied mathematics is dull and not real mathematics. In my opinion it is the other way around. I have worked in my time in many areas of pure mathematics: geometry, algebra, complex variable, probability theory, also mathematical statistics. All have exciting problems. So has mathematical programming, and what's more the latter is a hell of a lot more relevant. I am told that 25% of all "scientific computing" in the world involves math. programming.

With regard to the shortest route problem which I just cited, the problem is to characterize the mathematical properties of an optimal solution so that one can devise (what Jack Edmonds calls) a "good"

algorithm. For example, given m nodes and n arcs, can one devise a process that involves at most, say n additions and comparisons; if not, what is the best one can do? Is $m(m-1)(m-2)$ add and compare operations (which have been attained by some algorithms) the best one can do?

Now it turns out in the case of the shortest route problem that it has been thoroughly studied and that good algorithms have been devised. But this is only true if the arc distances happen to be non-negative. If the arc-distances can be negative, the entire theory collapses. The enlarged class of problems now includes the famous travelling salesman problem. This problem, incidentally, was kicking around in mathematical circles back in the 1930's long before anyone ever heard of O.R.

The problem was this: A travelling salesman has a sweetheart in the capital city of each of the 48 states. He feels it is his duty to arrange his tour so as to visit each one of his sweethearts. Find the shortest tour.

Mathematically this is a fascinating problem. The set of tours corresponds to the vertices of a certain convex polytope. Characterizing the faces of this polytope remains the unsolved problem.

Returning to my introductory remarks:

In my opinion, Operations Research should be viewed by mathematicians as a revitalizing force. A fertile ground for new, unsolved problems, new mathematical theories. Universities should be encouraged to develop a Division of Mathematical Sciences encompassing mathematics, operations research, statistics, computer science, and classical applied mathematics. Curricula should be revised so that all mathematics students get a good exposure to the mathematical sciences. I would recommend that the practice

of some mathematicians to downgrade the mathematical sciences be discouraged because: (1) it represents, on their part, ignorance, (2) lack of responsibility--but mainly I would discourage the practice because (3) it deprives the students of an exciting outlet for their talents.